Scattering fidelity in elastodynamics. II. Further experimental results

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(Received 17 March 2008; revised manuscript received 16 June 2008; published 18 December 2008)

The agreement of past measurements of elastodynamic scattering fidelity with predictions based on random matrix theory, even in bodies with ostensibly regular ray dynamics, has motivated further measurements with a precision capable of detecting fine deviations. Two aluminum blocks are studied, one rectangular, one irregular. As previously, it is found that fidelity decays more slowly in the irregular object. It is further shown that the time dependence of that decay corresponds well with predictions based on random matrix theory and the Gaussian orthogonal ensemble and that predictions based on Poissonian model statistics do not correspond. In some cases deviations from theory are seen that are consistent with partially broken reflection symmetries and inhomogeneous dissipation.

DOI: 10.1103/PhysRevE.78.066212 PACS number(s): 05.45.Mt, 43.35.+d, 03.65.Sq

INTRODUCTION

Recent measurements of scattering fidelity [1–4] as a function of the age of the waves in a reverberant cavity have compared well with predictions [4–6] from calculations assuming that the structure and its perturbations are well represented by the Gaussian orthogonal ensemble (GOE) of random matrices. Accordance has been observed in two-dimensional (2D) microwave cavities whose rays are chaotic [2,4] and in irregular and regular elastodynamic bodies [3]. Predictions of the *magnitude* of fidelity decay, as opposed to its time dependence, are based on an assumption that the rays mix well across phase space on every reflection from a boundary. These predictions were seen to correspond to measured fidelities in sufficiently irregular bodies but not in regular ones [1].

Here we revisit the elastic block systems discussed by Lobkis and Weaver [1] and analyzed further by Gorin *et al.* [3], and do so with particular attention to eliminating sources of systematic error and increasing statistical confidence and reliability. We make laboratory measurements that will permit to distinguish, more so than in [3], between different theoretical predictions. Two elastic blocks are studied. One is irregular with a corresponding expectation that its level statistics are well described by the GOE. Another is a rectangle with eight apparent reflection symmetry classes and ray paths corresponding to pseudointegrability and poor mixing even within one class. Each is perturbed by quasistatic variations in temperature, while reverberant elastodynamic responses are examined for change.

In the irregular body, fidelity between waveforms at different temperatures is found to diminish with age in the manner predicted by a theory based on GOE level statistics. With good confidence we conclude that a model of Poisson spectral statistics does not fit. The strength of the perturbation is found to be 50% greater than predicted by the simple theory. Slight deviations are observed that are perhaps ascribable to nontrivial dissipation statistics.

Fidelity's time dependence in the rectangular block is found, at higher frequencies, to be in accord with predictions from the GOE. A model of the block as having eight distinct reflection classes would suggest that the level statistics are those of eight independent spectra and thus well approximated by Poisson statistics. Thus the accord with the GOE is perhaps somewhat surprising. However, independent measurements of dissipation profiles strongly suggest that reflection classes mix at high frequency due to some undetermined symmetry-breaking mechanism, thus indicating that GOE statistics are the more relevant. Furthermore and for reasons described below, even if the reflection classes do not mix and the overall spectrum appears Poissonian, we would still expect a fidelity governed by the GOE. At very low frequencies where the symmetries are only slowly broken and where the dissipation statistics are extreme, we not surprisingly see a complicated time dependence that fits to neither existent theory. Magnitudes of the fidelity loss are compared with separate predictions and found, as seen and explained previously [1], to exceed simple theory by more than 100%.

SYSTEM

The experimental configuration is illustrated in Figs. 1 and 2. A transient piezoelectric pulse is applied to a 3-mm-diam transducer in oil-mediated contact with a block of aluminum alloy. Dry coupling would be less invasive and would not introduce inhomogeneous dissipation, but it lacks the robustness and sensitivity needed for measurements of

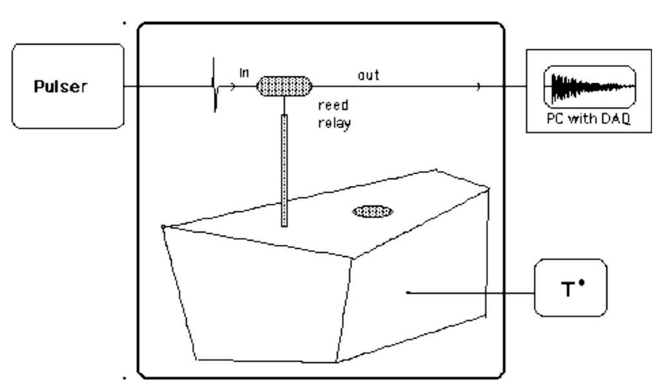

FIG. 1. Laboratory system. An aluminum specimen is allowed to cool in a vacuum as temperature and ultrasonic response are monitored. A reed relay isolates the response from the pulser.

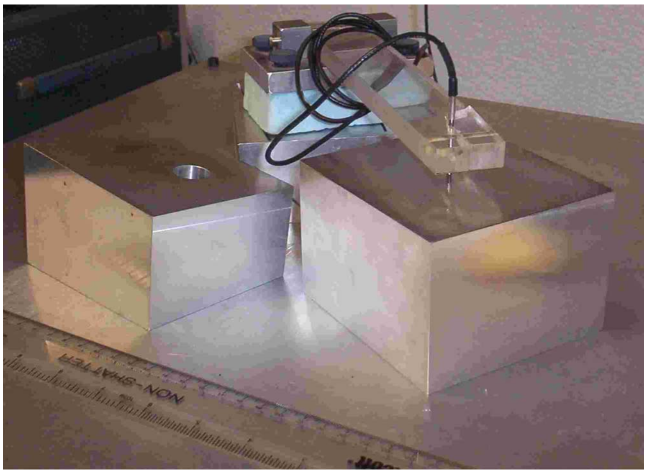

FIG. 2. (Color online) Photographs of the two aluminum blocks. The transducer is seen at the center of the $11.2 \times 10.0 \times 7.2\ \mathrm{cm}^3$ rectangular block.

fidelity. Reverberant response waveforms were taken, by the same transducer, while the specimen was allowed to cool from initial temperatures of about 55 °C to 27 °C. A reed relay isolated the receiving circuit from the pulser. Temperature in the irregular object was in some cases monitored by a thermocouple inserted in a small hole into the specimen, with precision of ±0.03 °C. In an attempt to preserve the reflection symmetries of the rectangular block no hole was drilled into that block, and temperature was estimated from readings taken in a neighboring sister block of the same size. The tests were conducted in a vacuum, thus eliminating convective, and most conductive, cooling mechanisms.

A useful first way to characterize reverberant ultrasound in these specimens is by examining the time dependence of the spectral power density in the reflected signal. Figure 3 does this for two cases, one of oil-coupled transducers on the irregular block at frequencies from 250 to 900 kHz and another of dry coupled transducers at the center, and off the center, of one face of the rectangular block at three low frequencies. Useful durations of the waveform were greater than seen previously [1,3]. Even after 200 ms, the signal strengths remained well above background noise. The quality factors Q are between 30 000 and 100 000.

Figure 3(a) shows the typical time dependence for the spectral energy density in the irregular block. The curves are essentially identical to those obtained with oil-coupled transducers at off-centered positions on the rectangular block. The decay is nominally exponential, but at 250 kHz there is curvature. Burkhardt [7] has shown how such a curvature can be enhanced in materials like the present specimens by inflicting localized damage. These nonexponential decays are ascribed to the existence of a finite number of distinct decay channels and a consequent variation in the decay rates of different modes, even at the same frequency. They have been studied systematically [7,8]. It is not clear from these data whether the distinct decay channels are intrinsic to the block or associated with the transducer. But the corresponding data from Fig. 3(b) for 250 kHz with a dry-coupled off-center transducer show no curvature. Comparison with Fig. 3(a) indicates that the curvature in Fig. 3(a) is due to losses into the transducer.

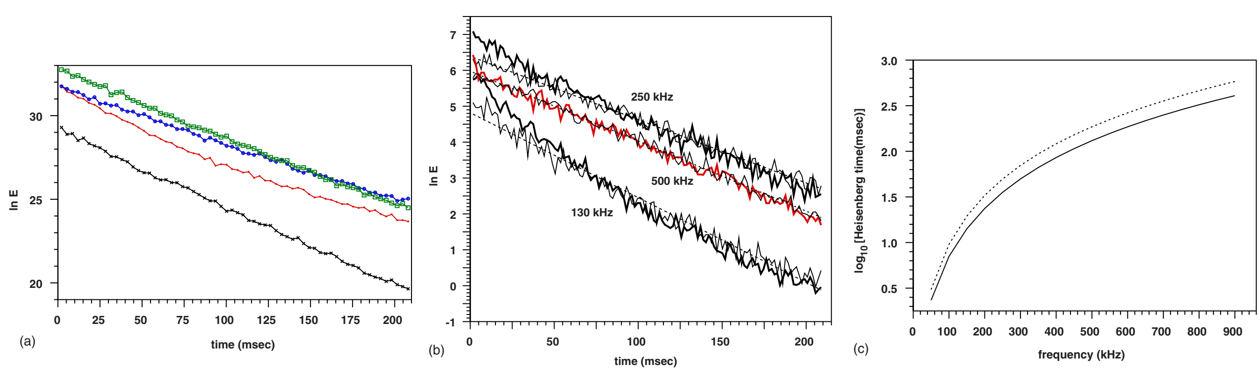

FIG. 3. (Color online) (a) The time dependence of the spectral power density in the signal received from the irregular block. E may be thought of as a measure of the local kinetic energy density; units are arbitrary. (×'s, 900 kHz; open squares, 650 kHz; large dots, 450 kHz; small dots, 250 kHz.) The typical signal duration is greater than 200 ms. Noise levels all lie below $\ln E = 12$ on this scale. Each point of the plot corresponds to a time window of duration 3 ms and a frequency bin of width 50 kHz. Fluctuations are of the expected order, of one part in $\sqrt{\Delta f \Delta t}$. As the quality factor Q is the number of cycles of free vibration before energy decays by a factor of $\exp(2\pi)$, these data can be used to discern the Q's. Thus $Q(f=25\ \mathrm{kHz})=37\,000$, $Q(f=450\ \mathrm{kHz})=82\,000$, and $Q(900\ \mathrm{kHz})=118\,000$. Data were taken with an oil-coupled transducer, so the curvature visible at 250 kHz could be intrinsic to the block or due to excess absorption at the transducer. That it is the latter may be concluded from comparison with (b) where an off-center dry transducer detects no curvature. (b) Energy decay profiles from three frequencies in the rectangular block. Heavy lines correspond to the transducer in the center of one face, light lines to an off-center position. Dashed lines are linear fits to the off-center data. The dependence on transducer position shows the finite time required to equilibrate between the laterally even-even modes excited by the centered transducer and to which it is sensitive and the other modes: even-odd, odd-even, and odd-odd. The effect is visible at and below 250 kHz. It disappears for higher frequencies. In order to isolate the effects of symmetry breaking from those of inhomogeneous dissipation, these data were taken with a small dry-coupled (and therefore noninvasive) transducer. Heisenberg times at these three frequencies are 186, 35, and 15 ms. (c) Heisenberg times from Eq. (1). The solid line corresponds to the irregular block, with $V=560\ \mathrm{cm}^3$ and $S=452\ \mathrm{cm}^2$. The dashed line corresponds to the rectangular block, with $V=808\ \mathrm{cm}^3$ and $S=530\ \mathrm{cm}^2$. Wave speeds were $c_s=0.315\ \mathrm{cm}/\mu\mathrm{s}$ and $c_p=0.635\ \mathrm{cm}/\mu\mathrm{s}$.

Figure 3(b) shows the spectral power densities from two different transducer positions and three different frequencies. A transducer in the center of one face is presumably only sensitive to those modes with lateral reflection symmetries that are *even*—that is, to only one mode in four. A transducer in an off-center position is sensitive to all modes with equal probability and is no respecter of the block's reflection symmetries. The spectral energy decay profiles for the off- and on-center transducers differ at low frequency. The initial decay appears faster when the transducer is centered, showing that the energy in the even-even modes is slowly scattered into the other modes where the transducer does not detect it. At higher frequency this effect diminishes; it is undetectable above 500 kHz, thus indicating that the symmetry breaking there is too fast to discern. A model of energy conduction [9] predicts a time-dependent spectral energy density $\sim 1 + 3\exp(-t/t_{leak})$. But such a model cannot apply if the leak time is comparable to or longer than the Heisenberg time [9]. In this case one would describe the modes as being partially dynamically localized into symmetry classes and expect an energy transport dynamics that cannot be modeled so simply.

The mechanism for scattering between symmetry classes has not been identified; calculations show that it cannot be due to the microscopic heterogeneities associated with aluminum crystallites [10]. Perhaps there is an inhomogeneous or anisotropic elastic texture. Perhaps the symmetry breaking is provided by the three needle supports. Whatever the mechanism, we conclude that our rectangular block is imperfectly symmetric and that waves are scattered between symmetry classes at a frequency-dependent rate. We further expect that low-frequency modes are partially localized. We also conclude that the oil-coupled transducers used to collect the fidelity data induce a degree of heterogeneous dissipation which will complicate analysis at low frequencies. The complicated energy transport dynamics seen at low frequencies suggests that comparisons between theoretical and measured fidelity, and especially its time dependence, are likely to be problematic.

Heisenberg times were estimated based on the asymptotic formula [11]

$$T_H = \frac{dN}{df} = 4\pi V f^2\left(\frac{2}{c_s^3} + \frac{1}{c_p^3}\right) + \frac{\pi S f}{2c_s^2}\left(3 + \frac{2c_s^4}{c_p^2(c_p^2 - c_s^2)}\right) + \cdots . \tag{1}$$

An additional term of the order of the edge length divided by the wave speed is not calculated. The fractional error induced by its neglect is small at high frequencies. Incorporations of plausible values for this term have been found, in the analyses reported below, to make no difference.

FIDELITY

As discussed elsewhere [1,12], a temperature difference leads to changes in longitudinal and shear wave speeds and an isotropic thermal expansion. If each wave speed changed with temperature in the same way, waveforms would only dilate with temperature change. But as the longitudinal and shear waves speeds have different temperature sensitivities, waveforms not only dilate in accordance with an average wave speed change and thermal expansion, but distort as well. This change of waveform shape, after correcting for the less interesting dilation, is termed a loss of scattering fidelity.

A correlation function is defined as [1]

$$X(\varepsilon) = \frac{\int S_{T_1}(t) S_{T_2}(t\{1+\varepsilon\})dt}{\sqrt{\int S_{T_1}^2(t)dt \int S_{T_2}^2(t\{1+\varepsilon\})dt}}, \tag{2}$$

where the time integration is over a time window centered on an "age" t and the signals $S(t)$ are taken at two different temperatures and filtered over narrow bands centered on various frequencies f.

In the absence of distortion, X will take a value of unity when ε takes a value corresponding to the degree of dilation. An example $X(\varepsilon)$ was plotted in [1]. X takes the form of a band-limited δ function, centered on a value of ε that scales with temperature. For narrow-band signals (here we use 50-kHz bins) it resembles a tone burst. The scattering fidelity is the value of X at its peak and is, in the presence of distortion, less than 100%. We define *distortion* $D(t,f) = -\ln X_{\max}$, $0 \le D < \infty$. X is essentially what others have defined as scattering fidelity [2–6]. It is a function of age t and frequency f. It varies with specimen and perturbation strength.

SIGNAL PROCESSING PROTOCOL

At intervals of time of the order of minutes, corresponding to changes of temperature of about 0.5 °C, ten successive waveforms were taken, each of duration 209.7 ms (from 0.5 ms after the pulse to 210.2 ms after), separated from each other by 200 ms dead times during which the data were transferred to PC memory before the next waveform was captured. The 0.5 ms initial dead time is imposed in order to allow the reed relay to finish switching. waveforms were captured at a digitization rate of 2.5 MSa/s, or $\delta t = 0.4$ μs, 2^{19} data points in all. Each waveform then received a time rescaling (interpolation is carried out by appeal to the Nyquist theorem and knowledge that the spectrum has no components above 1 MHz) to account for minor dilations between them due to the small differences of temperature during that 4 s. As distortion is proportional to the square of temperature change, it is safe to treat these closely separated waveforms as having a relative dilation, but no relative distortion. Each waveform was also shifted by an irregular amount, of the order of 0.2 μs, to correct for an uncontrolled jitter in the trigger times. Shifts and dilations were chosen to maximize the correlations between these ostensibly identical waveforms. The ten waveforms were then averaged. The procedure results in one averaged 200-ms waveform for each half degree from 55 to 27 °C, about 50 waveforms in all. Each test required a period of several hours.

For each of ~ 250 pairs of such averaged waveforms, comparisons were made by first imposing a shift to account for trigger jitter and then frequency-band-limiting each into 16 frequency bins from 140 to 900 kHz. The correlation X of Eq. (2) was then formed for each pair of interest and each frequency band and each time window and a value of ε extracted which maximized X. Time window durations of

4.2 ms were chosen to be less than Heisenberg times so as to be able to resolve changes, but great enough to avoid edge effects. Doubling or halving this number made no difference to the recovered distortions. The value of ε so obtained was used as a surrogate for temperature (it is more accurate than a thermocouple measurement and much more accurate than the guesses that were used on the rectangular specimen). For every pair separated by a temperature difference within 5% of a chosen value (20% for the block whose temperature could not be directly monitored) of 0.5 or 1.0 or 1.5… °C the distortions $D=-\ln X^{\max}$ were normalized by their different values of ε^2 (distortions are expected to scale with the square of the perturbation strength ΔT, in turn proportional to ε) and averaged. A reference temperature difference to assign to the set was then constructed by the observed scaling for aluminum [12], $\varepsilon=-2.8\times 10^{-4}$ per °C. Thus, for each of 10 temperature differences and for each frequency band f, we construct 45–50 values of distortion $D(t,f,\Delta T)$. These are averaged. One such set of measurements was conducted on the irregular block. Another test was conducted on the rectangular block with the transducer placed carefully in the center of one face. A third was conducted on the rectangular block, but with the transducer placed at an off-center position. The blocks were supported by three sharp metallic needlelike contacts. The blocks and transducer are pictured in Fig. 2.

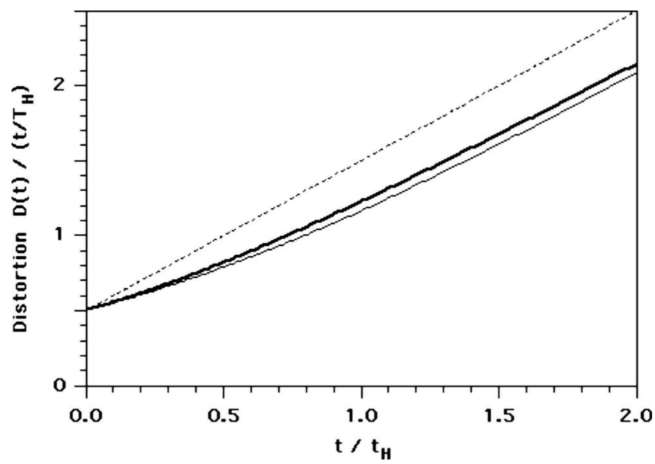

FIG. 4. Distortion $D=-\ln$, fidelity, at unit perturbation strength $\lambda=1$, divided by age, plotted out to twice the Heisenberg time. The three theoretical predictions, Eq. (3), differ only slightly: bold solid line, GOE; narrow solid line, GUE; dashed line, Poissonian. At much greater perturbation strengths where fidelity is tiny and distortion is high, the linear theory breaks down and there are further features; in particular, there is a predicted revival [17] near the Heisenberg time.

THEORY

Distortion is predicted to evolve with the age t of the waves since they were launched [2–6]. Assumptions that the specimen and the perturbation are well represented by a member of a Gaussian ensemble from random matrix theory (RMT), or by Poissonian modal statistics, predict that distortion has a time dependence

$$D(t) = \lambda^2[(t/2T_H) + (t/T_H)^2 - I(t/T_H)], \tag{3}$$

where

$$I(\tau) = \int_0^\tau dy \int_0^y b_2(x)dx$$

and where b_2 is the Fourier transform of the two-point level correlation function [13], equal to zero if the eigenfrequencies are uncorrelated. λ is a dimensionless measure of the perturbation strength. Equation (3) is plotted in Fig. 4 for the three cases of primary interest: GOE, Gaussian unitary ensemble (GUE), and Poissonian. The three theories make similar predictions; distinguishing among them demands a degree of measurement precision greater than that obtained earlier [1].

T_H is the Heisenberg time associated with those modes to which the initially excited modes couple through the temperature change. To the extent that our rectangular block has preserved its reflection symmetries, the temperature change will preserve them also and T_H should be one-eighth of the Heisenberg time of the entire block, regardless of the symmetry class(es) of the modes that are excited [14]. Similarly the modal statistics in b_2 should be those of the one-eighth part and very like those of the GOE [15]. Conversely, if waves are scattered between symmetry classes on a time scale fast compared to the Heisenberg time, then T_H should be that of the entire block and b_2 should again be that of the GOE. It is not clear what behavior is to be expected at intermediate degrees of symmetry breaking [16].

A theory for λ as proportional to temperature difference ΔT was presented in [1]. A ray-based picture was advanced in which bulk longitudinal and shear rays mode convert on reflection from the free surfaces and in which surface rays are neglected. On evaluating the fraction of energy that is mode converted at any specified angle of incidence and on assuming that each ray impinges at a random angle of incidence, Lobkis and Weaver estimated the mean lifetime of a P or S ray against mode conversion and, from that, estimated the variances of time spent as P or S. From that they deduced an expression for distortion proportional to the ratio of surface area to volume (i.e., depending on the rate of mode conversion), proportional to age t, and proportional to the square of the frequency:

$$D = Ct(\Delta T)^2(V/S)f^2. \tag{4}$$

The coefficient C depends on material properties. It was estimated to be, for aluminum, 3.26×10^{-4} per $^\circ\mathrm{C}^2$ per MHz^2 per ms per cm.

The theory purports to describe distortion at an early time where the ray picture is valid, a limit in which the value of T_H and the degree of symmetry are unimportant. Thus it may be used, after comparing to Eq. (3) at short times to predict the perturbation strength λ:

$$\lambda/\sqrt{2T_H} = \sqrt{\partial D/\partial t|_{t=0}} = \sqrt{C}(\Delta T)(V/S)^{1/2}f. \tag{5}$$

For the irregular block $\lambda/\sqrt{T_H}$ is predicted to scale with temperature and frequency by a proportionality of 0.0201 in-

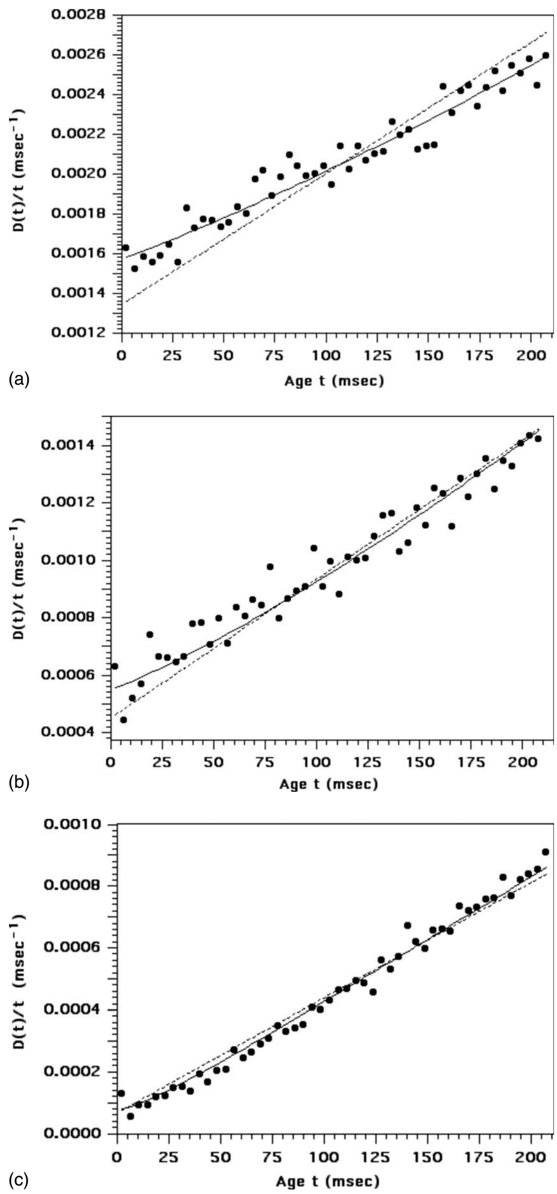

FIG. 5. (a) The measured distortion in the irregular block at 900 kHz and $\Delta T=2$ °C, divided by age in ms, is compared to the predictions of the Poissonian (dashed line) and GOE (solid line) models, each with λ as an adjustable parameter. Heisenberg time is 407 ms. (b) The measured distortion in the irregular block at 600 kHz, divided by age in ms, is compared to the best fits to the Poissonian (dashed line) and GOE (solid line) models. $\Delta T=2$ °C. Heisenberg time is 185 ms. (c) The measured distortion in the irregular block at 250 kHz, divided by age in ms, is compared to the predictions of the Poissonian (dashed line) and GOE (solid line) models. The very small but statistically significant deviations from theory—e.g., the consistent deficit between 25 and 75 ms—are intriguing. It may be that the inhomogeneous dissipation which generated the curvature of the decay profile seen in Fig. 3(a) is also responsible for this behavior. $\Delta T=2$ °C. Heisenberg time=35 ms.

verse root ms per MHz per degree. For the rectangular block the predicted proportionality is greater, 0.0223, owing that block's smaller surface-to-volume ratio.

RESULTS: IRREGULAR BLOCK

Distortions were found to have an unexpected small finite value at time zero. The zero-time distortion was an irregular function of frequency and was ascribed to the temperature dependence in the transducer response [18]. Theory indicates that the effect should be additive, and so we have subtracted the $t=0$ intercept of $D(t)$ in order to plot D/t in Fig. 5. This figure shows distortion in three different frequency bins, each of width 50 kHz.

The plots also show the data's best fit to the one-parameter (λ) models [Eq. (3)] with T_H taken from Eq. (1). The GOE predictions match the measurements. It is furthermore clear that Poissonian theories do not match the data. Earlier work [3] was unable to make this distinction, as the precision of the measurements of Lobkis and Weaver [1] did not permit it. Gorin *et al.* [3] instead merely noted that the GOE theory fit well, was consistent with RMT, and so permitted a better estimate of λ than had been made by Lobkis and Weaver.

At 250 kHz, Fig. 5(c), one discerns small differences between theory and measurements. We conjecture that the discrepancy is attributable to the nonuniform distribution of dissipation responsible for the curvature in Fig. 3(a). An extension of the theory [6] to systems with modes having a distribution of decay rates [7] has been made and a modified equation (3) has been derived. If there is a distribution of modal amplitude decay rates, $dP=p(\gamma)d\gamma$, γ being measured in units such that the Heisenberg time is unity, then we replace $I(\tau)$ with

$$I(\tau)=\int p(\gamma)e^{-\gamma\tau}d\gamma\int_0^\tau dy\int_0^y b_2(x)\bar{p}(x)e^{\gamma x}dx/\bar{p}(\tau)\bar{p}(0), \tag{6}$$

where $\bar{p}$ is the Laplace transform of p. Energy decay rates such as those seen in Fig. 3(a) are given by $E(t)=E_0\bar{p}(2t)$.

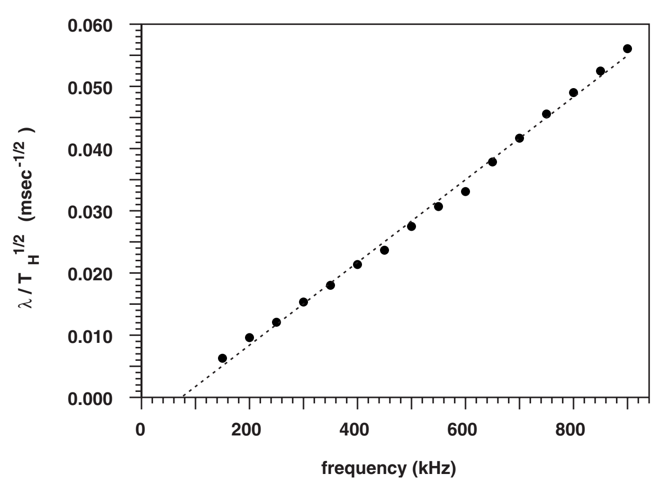

FIG. 6. The rate of fidelity decay at early times at $\Delta T=2$ °C, as recovered from the fits. This was predicted [1] to be proportional to frequency; see Eq. (5). The lowest three points are the least reliable, as effects similar to those discussed in Fig. 5(c) may have affected the best-fit value of λ. The dashed line is a linear fit. The observed slope is 0.068, greater than the theory, 0.0402 A finite intercept was not predicted by theory [1].

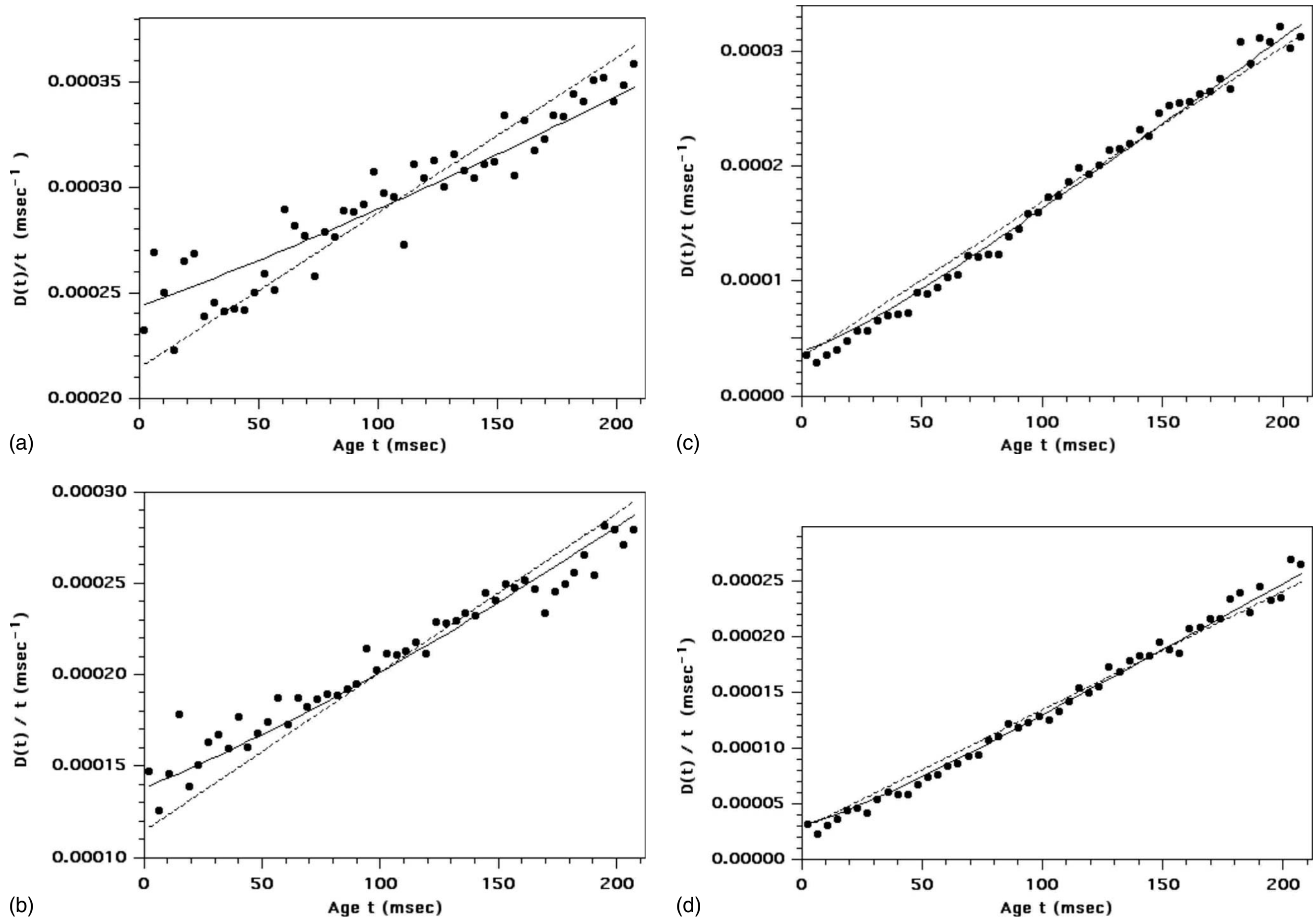

FIG. 7. (a) The distortion in the regular rectangular block behaves at 900 kHz as it does in the irregular block, matching the GOE predictions better than the Poissonian ones. $\Delta T=0.46°$. Theoretical predictions were based on the Heisenberg times of the entire block [Fig. 3(c)]. The data would not fit a prediction based on an assumption that the wave fields were superpositions of waves of distinct reflection symmetries. This plot is for the case of the transducer in center of the block's top face, but a similar plot for an off-center position does not differ significantly. Heisenberg time is 582 ms. (b) Distortion at 600 kHz, $\Delta T=0.46°$. Transducer in the center of the topside of a rectangular block. Heisenberg time is 264 ms. (c) Distortion at 250 kHz, $\Delta T=0.46°$. Transducer in the center of the topside of a rectangular block. Heisenberg time is 49 ms. We observe the same slight deviation from theory seen in Fig. 5(c). (d) Distortion at 250 kHz, $\Delta T=0.39°$. Transducer off-center on a rectangular block. Heisenberg time is 49 ms.

The value of λ taken from the fits is plotted versus frequency in Fig. 6. The ratio $(\lambda/f\sqrt{T_H})$, related to the initial rate of distortion, dD/dt, is comparable to that given by the simple theory [Eqs. (4) and (5)]. The low-frequency behavior of this ratio does, however, not comport with the simple theory. The difference does not disappear if the analysis is redone with plausible different values of T_H. But the theory was based on a picture of the ultrasonics being in the form of rays in three dimensions. At low frequencies the ultrasonic field is increasingly dominated by surface waves [19]. The contributions of the bulk and surface waves become equal at a frequency of the order of $f=Sc_T/4V\sim 45$ kHz. The apparent intercept seen in Fig. 6 is of this order, and so one conjectures that the discrepancy with theory may be due to its neglect of surface effects.

In sum, fidelity in the irregular block matches well to the predictions of RMT and not to Poissonian models, although there remain some uncertainties at longer wavelengths where the theories are almost indistinguishable and where analysis requires accounting for inhomogeneous dissipation.

RESULTS: RECTANGLE

Distortions in the regular rectangular block are plotted in Fig. 7. For frequencies above 250 kHz the measurements match well to the predictions of the GOE, but not to predictions based on an assumption of Poissonian eigenstatistics. This is the case whether the transducer is in the center of the block's top face or not. While the block has an ostensible eight-fold symmetry, the GOE fits nonetheless. The fits require the use of the Heisenberg time associated with the entire block, not just the that of the even-even modes. This is consistent with the earlier observation, based on Fig. 3, that the different symmetry classes mix rapidly at these frequencies and that the block's spectrum is not a superposition of eight independent spectra.

At 250 kHz, fidelity from centered and off-center transducers [Figs. 7(c) and 7(d), respectively] looks about the same. Both fit to theory at least as well as did Fig. 5(c). That they are so similar to Fig. 5(c) argues that their behaviors are of similar origin. But closer analysis shows that distortion is

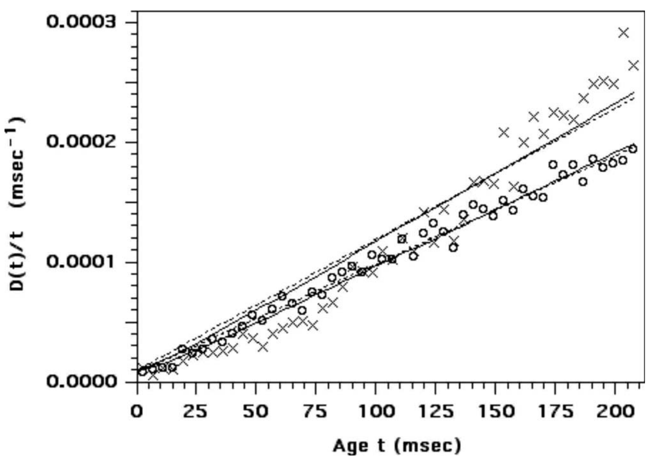

FIG. 8. Distortion for 140 kHz in the regular block with an off-center transducer, $\Delta T=0.39°$ (open circles) and for an on-center transducer (×'s). Heisenberg time is 16 ms. Respective fits to GOE and Poissonian models are given by the lines.

much greater, relative to the different values of ΔT^2, in the regular block. According to Eq. (3), D/t at times $t \gg T_H$ is $\{\lim_{t=0}(2D/t)\}\ t/T_H$. Either the perturbation as quantified by $\lim_{t=0}(2D/t)$ is much stronger in this block than in the irregular block or the effective Heisenberg time is less. It is apparent that Figs. 7(c) and 7(d) could be fit equally well if one chose a smaller value of T_H corresponding to imperfectly broken reflection symmetries; the consequent value for λ would be very different.

Figure 8 shows fidelity at a low frequency, 140 kHz, for which quantitative comparisons with theory are highly problematic. After allowance for different ΔT, fidelity at late times is independent of transducer position; the same Heisenberg time and $\lim_{t=0}(2D/t)$ apply to both cases. But the transducer position does affect fidelity at short times. Here there is a difference between fidelity [4] and scattering fidelity [2,3]; the behavior depends on initial state. For the off-center transducer (open circles), the fits (done with the choice $T_H=16$ ms) are good, but a choice of a shorter T_H (e.g., one-eighth of 16 due to eight independent symmetry classes) would lead to fits that were as good, but with different values for $\lim_{t=0}(2D/t)$. Fidelity for a centered transducer (×'s) has peculiar short-time behavior. It differs from the case of an off-center transducer in a manner reminiscent of the difference in spectral decay profiles of Fig. 3(b). A detailed analysis would require a theory of fidelity decay in systems with partial localization and perhaps also inhomogeneous dissipation. It is not attempted here. It is merely noted that the time scales of the anomalous behaviors are the same in both plots, Figs. 8 and 3(b), thus suggesting similar physical origins.

Perturbation strengths λ recovered from the regular block show a frequency dependence (Fig. 9) with a slope and an intercept different from that for the irregular block (Fig. 6). The ratio $\lambda/T_H f$ at high frequencies and after normalizing by ΔT is greater than it is for the irregular block. The difference was seen in [1] also and explained there as a failure of the theoretical model that assumed rays impinge at angles of incidence independent of their character as longitudinal or

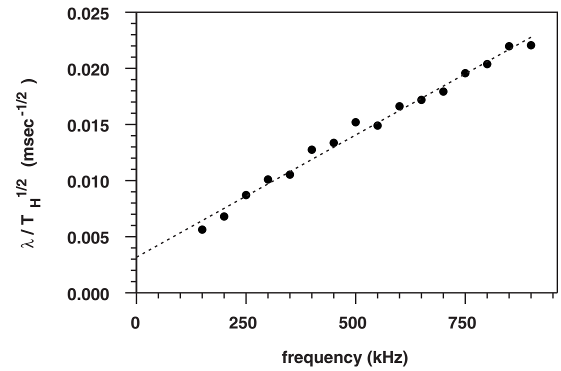

FIG. 9. Recovered perturbation strengths for the regular rectangular block at $\Delta T=0.46°$ with transducer in the center are approximately proportional to frequency, as predicted. As expected, the strength of the perturbation is different for the two blocks. The ratio at 900 kHz is now 0.0244, more than twice that predicted (0.0103 $=0.0223$ times 0.46°). As previously, this is ascribed to the failure of the simple theory to account for correlations among the rays in a regular block. The nonzero intercept is unexplained, but we note that, were smaller values of T_H to be chosen in the fits for frequencies at and below 250 kHz, the fits would have been as good, the three lowest points on this plot would be lower, and the intercept would be rather different.

shear and of their history. This assumption is poorly justified in regular objects, so as previously seen, distortion is stronger in the regular bodies. This figure portrays the values of λ as extracted from best fits using Heisenberg times for the entire block. As argued above, this is questionable at low frequencies where the modes partly localize in reflection classes. Had we chosen instead Heisenberg times associated with a subset of the modes, then the three leftmost data points in Fig. 9 would be lower by factors of the square root of the ratio of the Heisenberg times used in the fits.

SUMMARY

Measurements of scattering fidelity in an irregular block are found to fit predictions from random matrix theory and the Gaussian orthogonal ensemble. Predictions based Poissonian level statistics differ only slightly from those of the GOE, but the data do not support them. The lack of correspondence is statistically significant. This is as expected. Measurements at short wavelengths in an ostensibly regular rectangular block also fit well to the GOE predictions and not to the Poissonian. This was not expected, but independent measurements show that the rectangular block is imperfectly symmetric; mixing between symmetry classes precludes application of Poissonian statistics. At low frequencies in the rectangular block there are indications that slow mixing between reflection symmetry classes and inhomogeneous dissipation leads to a different character for fidelity's time dependence.

Perturbation strengths are not predicted by RMT. A short-wavelength theory [1] based on rapid mixing between bulk longitudinal and transverse waves at surface reflections predicts strengths that are proportional to frequency and comparable to those measured. Differences between theory and measurements for the frequency proportionality are observed and comprehended as previously [1].

ACKNOWLEDGMENTS

We thank Thomas Seligman and Thomas Gorin for discussions. The work was supported in part by the NSF, Grant No. CMS 05-28096.

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[15] It is perhaps not widely appreciated that ostensibly regular elastodynamic bodies, when attention is confined to an irreducible symmetry class, exhibit what appear to be GOE level statistics unlike scalar waves [K. Schaadt, A. P. B. Tufaile, and C. Ellegaard, Phys. Rev. E **67**, 026213 (2003)]. This is in part due to mode conversion at free surfaces and in part to the strong diffraction at corners and edges.

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